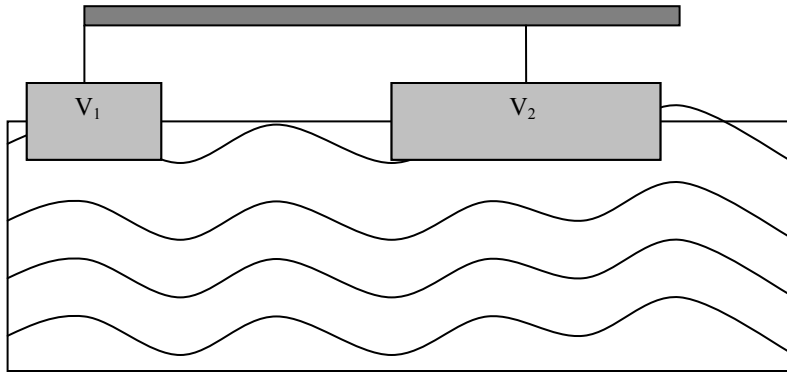
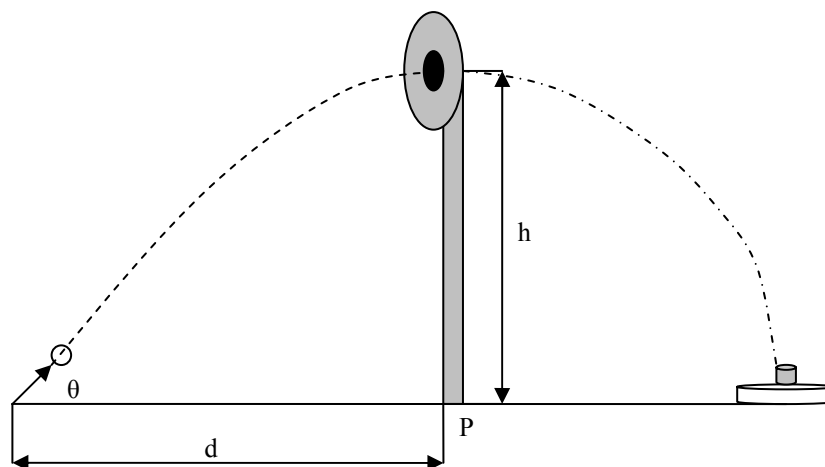


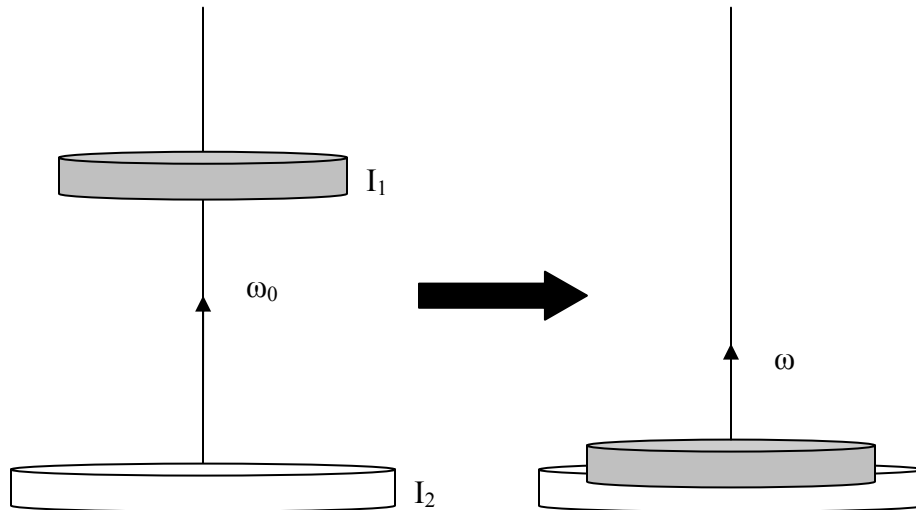
1. A deck of mass  $M$  is suspended above water by 2 pontoons (of different sizes) whose density is  $\frac{1}{4}$  that of water. The pontoons are both halfway submerged below the water. They are attached to the platform at one end and at  $\frac{1}{4}$  the length of the deck from the other end, as shown in the figure. Find  $V = V_1 + V_2$ , the sum of the volumes of the two pontoons. Find the individual volumes  $V_1$  and  $V_2$  of each of the pontoons.



2. In a carnival game the objective is to throw a ball of mass  $m$  and hit a target on a pole, tipping it over. If the end of the pole then strikes a button with enough speed, a buzzer goes off and you win a prize. The ball and the target are covered in Velcro so they stick together. The pole is of height  $h$ , mass  $M$ , and hinged at one end so it can rotate about point  $P$ . You figure your best chance of winning the game is to make the ball hit the target with all of its velocity in the horizontal direction. If you are standing a horizontal distance  $d$  from the pole and throw the ball with a launch velocity of  $v_L$ , at what angle  $\theta$  must you throw the ball? About point  $P$ , what is the ratio of the angular momentum when you throw the ball,  $L_{\text{throw}}$ , to the angular momentum when the ball hits the target,  $L_{\text{target}}$ , if  $d=h$ ? What is the velocity of the end of the pole as it strikes the button? The moment of inertia of the pole about its center of mass is  $\frac{1}{12} ML^2$ .

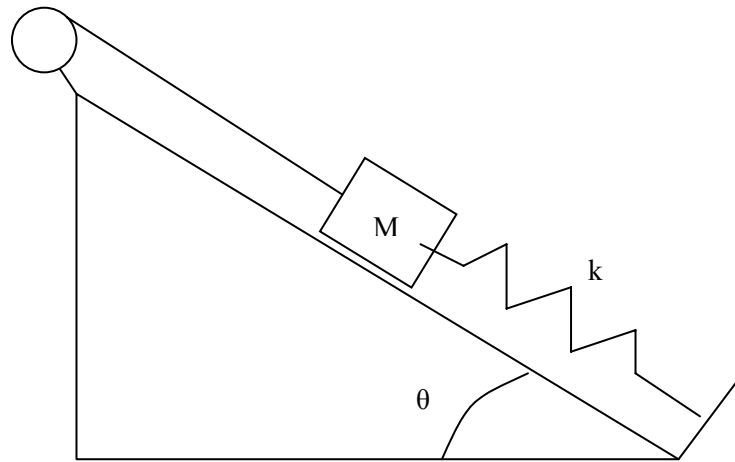


4. A disk with moment of inertia  $I_1$  rotates on a vertical frictionless axle with angular velocity  $\omega_0$ . A second disk of mass  $M$  and radius  $R$  with moment of inertia  $I_2$  drops onto the first. The coefficient of friction between the two surfaces is  $\mu_k$  and the two disks eventually reach the same angular velocity  $\omega$ . Calculate  $\omega$ . Is energy lost in this situation? Find the ratio of the system's final to initial rotational energy. Find the total torque acting on each disk. Show that the two disks reach the angular velocity  $\omega$  at the same time.

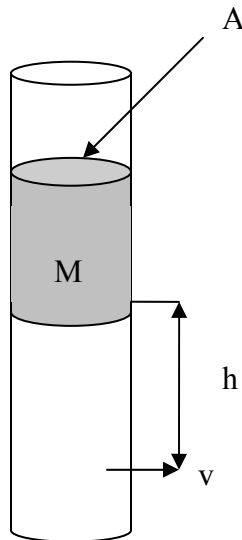


5. The pulley shown in the figure has mass  $m$  and radius  $R$ . One end of a block with mass  $M$  is attached by a string to the pulley. The other end is attached to a spring with spring constant  $k$ . The pulley axle and incline of angle  $\theta$  are frictionless, but there is friction between the string and pulley. The pulley is wound counterclockwise until the spring is stretched a distance  $d$  from equilibrium, then released. What is the angular acceleration of the pulley as a function of the spring's distance from equilibrium? When the spring reaches the unstretched position, what is the angular velocity of the pulley? After the mass is released from rest, it acts as a Simple Harmonic Oscillator. Write an equation for the position of the block's center

of mass as a function of time.



6. A mass  $M$  just barely fits inside a water-filled cylinder with cross section  $A$  and makes a watertight, but frictionless seal. A small hole one-tenth the area of the tube exists a distance  $h$  below the mass. What is the velocity of water leaving the hole?



7. From measurements of the speed of stars very near the center of our Milky Way Galaxy, it has been determined that there is a mass of  $2.6 \times 10^6$  times the mass of our sun,  $M_s$ . If this dark object is a supermassive black hole as some astronomers suggest, calculate the Schwarzschild radius of this black hole (defined as the radius the mass would shrink to where even light could not escape).

The Sun is situated a distance  $D$  (estimated at 30,000 ly) about halfway out to the edge of the galaxy. The measured velocity of the Sun's rotation about the center of the galaxy is

$v$  (about 250 km/s). Find an expression for the period of the Sun's rotation about the center of the galaxy. Determine the mass of material in the galaxy within the Sun's orbit,  $M_G$ .

The Sun is situated near the outer edge of where visible stars are found (mass in the outer galaxy is mainly dark matter). If stars account for all the mass of the galaxy within the Sun's orbit, except for the black hole at the center, and the Sun is a star of average mass, write an expression to estimate the number of stars in the Milky Way Galaxy.

